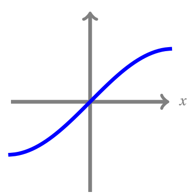
Worksheet Eight **MATH 104** Name:____________________

WORK ON PROBLEMS IN GROUP OF 2-4. YOUR INSTRUCTOR WILL MARK YOUR GROUP WORK IN CLASS. TURN IN YOUR OWN WORK FOR QUESTIONS MARKED AS "INDIVIDUAL WORK" INDIVIDUALLY; UPLOAD TO CANVAS OR SUBMIT IN CLASS ON THE DUE DATE.

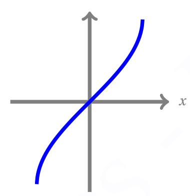
6.3: Inverse Trigonometric Functions

- Graphical Approach: We learned that sine, cosine and tangent functions are periodic so they are not one-to-one. To define an inverse function for them, we restrict their domain to intervals that contains the largest one-to-one piece of their graph. The following are the standard form of these restrictions.

$y = \sin(x)$ restricted to domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



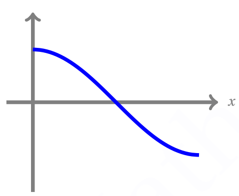
$y = \arcsin(x)$



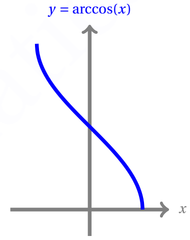
Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = \cos(x)$ restricted to domain $[0, \pi]$



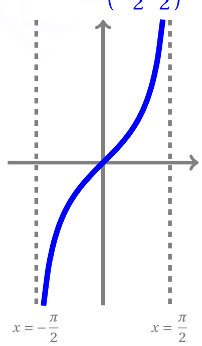
$y = \arccos(x)$



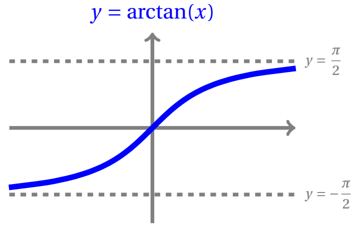
Domain: $[-1, 1]$

Range: $[0, \pi]$

$y = \tan(x)$ restricted to domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



$y = \arctan(x)$

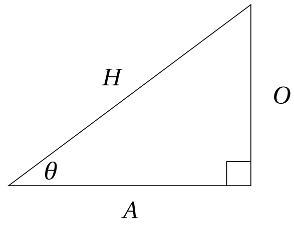


Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

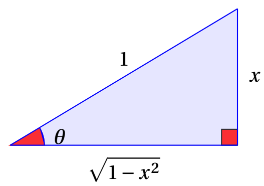
Now, you can complete Problem 1.

- Inverse trigonometric functions and right triangles:



$$\sin(\theta) = O/H \qquad \arcsin(O/H) = \theta$$
$$\cos(\theta) = A/H \qquad \arccos(A/H) = \theta$$
$$\tan(\theta) = O/A \qquad \arctan(O/A) = \theta$$

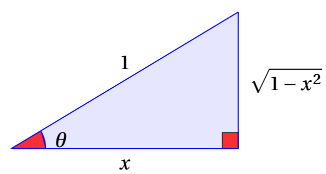
- Right triangles can be used to find sine, cosine and tangent of an inverse trig function:

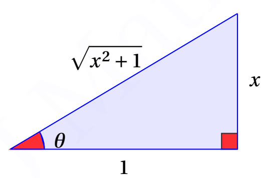


- $\sin(\arcsin(x)) = x$
- $\cos(\arcsin(x)) = \sqrt{1-x^2}$
- $\tan(\arcsin(x)) = \dfrac{x}{\sqrt{1-x^2}}$

Now, you can complete Question 2 and 3.

- $\sin(\arccos(x)) = \sqrt{1-x^2}$
- $\cos(\arccos(x)) = x$
- $\tan(\arccos(x)) = \dfrac{\sqrt{1-x^2}}{x}$





- $\sin(\arctan(x)) = \dfrac{x}{\sqrt{x^2+1}}$
- $\cos(\arctan(x)) = \dfrac{1}{\sqrt{x^2+1}}$
- $\tan(\arctan(x)) = x$

Now, you can complete Question 5.

- Note:

1. $\sin(\arccos(x)) \geq 0$, $\cos(\arcsin(x)) \geq 0$ and $\cos(\arctan(x)) \geq 0$ even if x is negative. This can be explained using the range of arc functions and quadrants of unit circle.
2. $\sin(\arctan(x))$, $\cos(\arccos(x))$, $\sin(\arctan(x))$, $\sin(\arcsin(x))$, and $\tan(\arctan(x))$ have the same sign as x.
3. By (1) and (2) and using the unit circle, the previous formulas are true for any x.

- Other notations for inverse functions: $\sin^{-1}(x) = \arcsin(x)$, $\cos^{-1}(x) = \arccos(x)$ and $\tan^{-1}(x) = \arctan(x)$. Note that the inverse function notation is only true for the restricted domain. Note, $^{-1}$ does not mean reciprocal; $^{-1}$ means inverse.
  For example, $\tan^{-1}(x) \neq \dfrac{1}{\tan(x)}$.

- Warning: Even though $\sin(\arcsin(x)) = x$, $\cos(\arccos(x)) = x$ and $\tan(\arctan(x)) = x$, the other way around for these compositions does not always work.
  For example, $\arcsin\left(\sin\left(\dfrac{3\pi}{4}\right)\right) \neq \dfrac{3\pi}{4}$, $\arccos\left(\cos\left(\dfrac{5\pi}{4}\right)\right) \neq \dfrac{5\pi}{4}$ and so on.

- However, if the domain is restricted to $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$, then $\arcsin(\sin(x)) = x$.
  If the domain is restricted to $[0, \pi]$, then $\arccos(\cos(x)) = x$.
  If the domain is restricted to $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$, then $\arctan(\tan(x)) = x$.

- When finding arc functions, it is very important to remember the following formulas:
  - Note that $\theta = \arcsin(x) \implies x = \sin(\theta)$
  - Note that $\theta = \arccos(x) \implies x = \cos(\theta)$
  - Note that $\theta = \arctan(x) \implies x = \tan(\theta)$

- The steps in finding composition: When finding $\text{trig function}\left(\text{arc function}\left(\text{x}\right)\right)$, follow these steps:
  1. $\text{trig function}\left(\underbrace{\text{arc function}(x)}_{\theta}\right) \implies \text{trig function}(\theta) = x$
  2. Draw a right triangle with angle θ such that trig function of θ is x. Find the other trig functions.
  3. Find trig function(θ).

- Note the input of a trig function such as sine, cosine,... is an angle in radian or degree and the output is a ratio of two lengths.

  input(Measure of an Angle) ➔ $\boxed{\text{Trig}}$ ➔ output(Ratio)

- On the other hand, the input of an arc function is a ratio and the output is an angle.

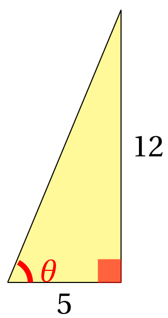
  input(Ratio) ➔ $\boxed{\text{Arc}}$ ➔ output(Measure of an Angle)

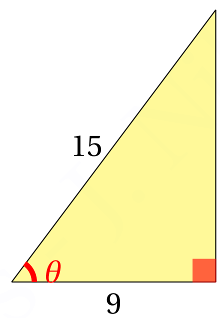
Now, you can complete Problems 5 and 6.

1. What is the domain and the range of each of the following arc functions?

   (A) $y = \arcsin(x)$ (B) $y = \arccos(x)$ (C) $y = \arctan(x)$

2. Find the relationship between θ and the given lengths in each triangle; find θ using your calculator arc functions.





3. Write $\tan(\sin^{-1}(x))$ as an algebraic expression in terms of x.

   (a) $\dfrac{x}{\sqrt{1-x^2}}$

   (b) $\dfrac{1}{x}$

   (c) $\dfrac{x}{1-x}$

   (d) $1-x^2$

4. Find each of the following trigonometric values; when needed, use a triangle in each case.

(a) $\tan\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$

(b) $\tan\left(\tan^{-1}\left(\frac{4}{3}\right)\right)$

(c) $\sin^{-1}(4)$

(d) $\tan(\arctan(1.7))$

(e) $\sin(\arctan(1))$

(f) $\cos\left(\arcsin\left(\frac{\sqrt{2}}{2}\right)\right)$

(g) $\cos\left(\arcsin\left(\frac{\sqrt{3}}{2}\right)\right)$

(h) $\cos\left(\arcsin\left(\frac{-1}{2}\right)\right)$

(i) $\sin\left(\arccos\left(\frac{-3}{5}\right)\right)$

5. Evaluate the following; noting that range of $\arcsin(x)$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, the range of $\arccos(x)$ is $[0,\pi]$ and the range of $\arctan(x)$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

(First compute the inside function, then make sure that the output of the arc functions are in the ranges mentioned.)

(a) $\arccos\left(\cos\left(\frac{7\pi}{6}\right)\right)$

(b) $\arcsin\left(\sin\left(\frac{7\pi}{6}\right)\right)$

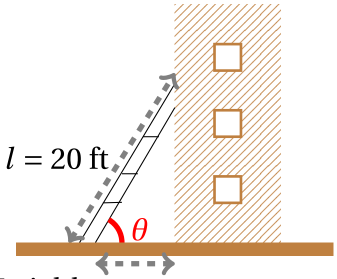
(c) $\arccos\left(\cos\left(\frac{5\pi}{6}\right)\right)$

(d) $\arcsin\left(\sin\left(\frac{5\pi}{6}\right)\right)$

(e) $\arcsin\left(\cos\left(\frac{7\pi}{6}\right)\right)$

(f) $\arccos\left(\sin\left(\frac{7\pi}{6}\right)\right)$

6. Katheryn is observing a 20-ft ladder leaning against a building. **The first time she notices the ladder**, the base of the ladder is 16 ft from the base of the building. She notices that the the ladder is sliding away and the base is **increasing** at a constant rate $4 ft/sec$.



(A) Find the angle between the ground and the ladder, θ, at the moment Katheryn notices it.

(B) Express x, the distance of the base of the ladder from building at any time $t\,sec$, as a function of t. (In terms of t.)

(C) Express θ as a function of time $t\,sec$; noting the arc functions. (In terms of t.)

INDIVIDUAL WORK

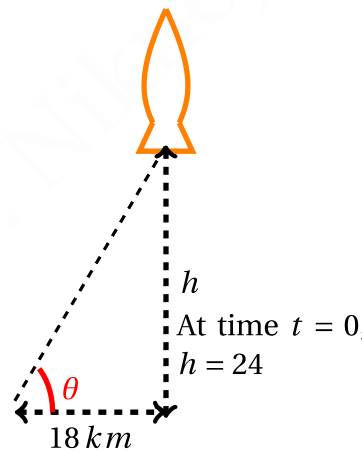
UPLOAD TO CANVAS OR SUBMIT IN CLASS BEFORE DUE DATE. DISCUSSING THESE QUESTIONS IN YOUR GROUP IS ENCOURAGED BUT MAKE SURE YOU ARE TURNING IN YOUR OWN WORK.

7. Melody is observing the space shuttle from a **constant** distance of 18 km from the launch pad. The shuttle's vertical speed is 11.3 km/sec and when Melody first start looking at the shuttle, it is 24 km above the lunch pad.

   (A) *(0.5 points)* Express the angle of elevation θ as a function of the height of the space shuttle h, in kilometers.

   (B) *(1 point)* Express h as a function of time t, in seconds.

   (C) *(1 point)* Express θ as a function of time t.



8. *(1.5 points)* Evaluate the following.

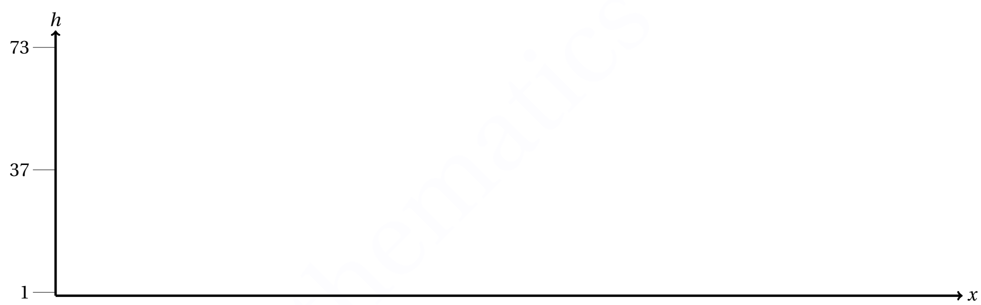
   (A) $\sin\left(\arccos\left(\frac{8}{17}\right)\right)$ (B) $\tan\left(\arccos\left(\frac{8}{17}\right)\right)$ (C) $\arcsin\left(\sin\left(\frac{11\pi}{6}\right)\right)$

9. **Refer to Section 6.1:** A Ferris wheel's radius is 36 meters and boarded from a platform that is 1 meter above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 6 minutes. The function $h(t)$ gives a person's height in meters above the ground t minutes after the wheel begins to turn.

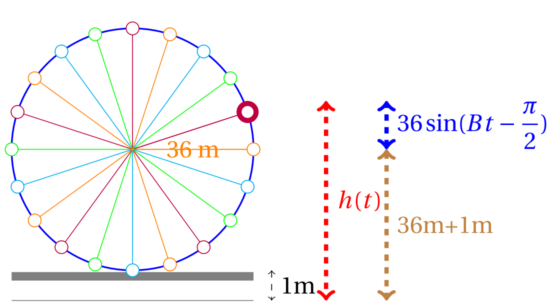
(A) *(1.5 points)* Find the **amplitude**, midline, and **period** of $h(t)$.

(B) *(1 point)* Find a sine formula for the height function $h(t)$.

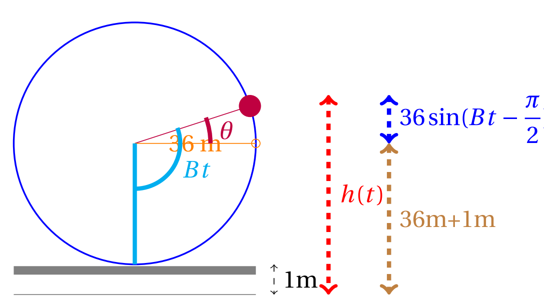
(C) *(0.75 points)* Graph the function in Part (B) noting Part (A); include at least one full period.



(D) *(0.75 points)* How high off the ground is a person after 7 minutes?



https://www.geogebra.org/m/sn9hrdhp



Videos:

- https://mediahub.ku.edu/media/t/1_wmpyodwx
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